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Directed Cooperation of Multi-Agent Systems

Introduction of Distributed Algorithms for Cooperative Control of Multi-Agent Systems over Directed Graphs

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To Ren and Akiko

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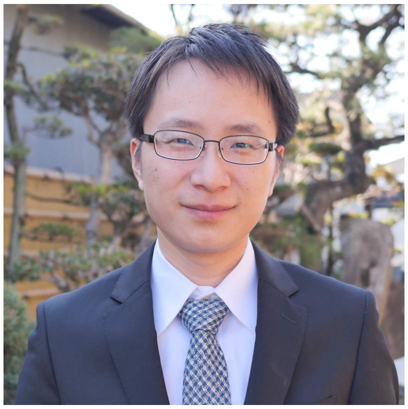

Kai Cai received the B.Eng. degree in Electrical Engineering from Zhejiang University, Hangzhou, China, in 2006; the M.A.Sc. degree in Electrical and Computer Engineering from the University of Toronto, Toronto, ON, Canada, in 2008; and the Ph.D. degree in Systems Science from the Tokyo Institute of Technology, Tokyo, Japan, in 2011. He is currently a Full Professor at Osaka Metropolitan University. Previously, he was an Associate Professor at Osaka City University (2014–2020), an Assistant Professor at the University of Tokyo (2013–2014), and a Postdoctoral Fellow at the University of Toronto (2011–2013). Dr. Cai's research interests include cooperative control of multi-agent systems, discrete-event systems, and cyber-physical systems. He is the co-author (with W.M. Wonham) of Supervisory Control of Discrete-Event Systems (Springer 2019) and Supervisor Localization (Springer 2016). He is serving as an Associate Editor for IEEE Transactions on Automatic Control and a Senior Editor for Nonlinear Analysis: Hybrid Systems. He was the Chair for IEEE CSS Technical Committee on Discrete Event Systems (2019–2022), and a member of IEEE CSS Conference Editorial Board (2017–2022). He received the Pioneer Award of SICE in 2021, the Best Paper Award of SICE in 2013, and the Best Student Paper Award of IEEE Multi-Conference on Systems & Control in 2010.

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Preface

Cooperative control of multi-agent systems has been actively studied in the field of systems and control in the past two decades. Such systems typically consist of a large number of distributed agents, which locally interact with one another such that they jointly pursue a global goal. Research results on cooperative control of multi-agent systems have found wide applications in robotics (swarms of vehicles/drones) [CWRKG20, MC19, SVC+16], engineering (sensor/power networks) [CAYM15, DB10, OS07], physics (systems of oscillators) [DCB13, PR11, SS08], epidemics (spreading processes) [YLAC21, KBG14, OGNK13], and social/political science (opinion dynamics) [YLA+18, FJB16, AL15]. The literature has grown in near-intractable volumes, but excellent textbooks (e.g. [Bul22, FM16, BAW11, ME10, RB08]) and surveys (e.g. [OPA15, DB14, CYRC13, GS10, OSFM07]) have kept the content in organized manners.

In writing this book, we aim to provide a new perspective to link together various research work on cooperative control of multi-agent systems. This perspective is on different types of *graph Laplacian matrices*. The standard (conventional) Laplacian matrix is defined based on a nonnegative *adjacency matrix* [Bap10, GR00], which describes the interaction graph topology of a multi-agent system. This type of Laplacian matrix is fundamental in describing the dynamics of a number of multi-agent cooperative control problems including consensus, averaging, synchronization, regulation, flocking, and optimization [JLM03, INK19, CI11, CI12, Ren08, Lun12, WSA11, KCK20, OS06, XHC+17, ZYC20]. The algebraic properties of this type of Laplacian matrix have been found to characterize stability and performance of the corresponding cooperative control algorithms. These algebraic properties are also closely related to the connectivity properties of the interaction graph.

More recently, two other types of Laplacian matrices have been proposed in designing cooperative control algorithms. One type is defined from a complex-valued (entry-wise) adjacency matrix, and is called *complex Laplacian*. A complex Laplacian matrix has been found useful in solving a class of formation control and localization problems on a 2D space (that can be represented as a complex plane) [LDY+13, LWHF14, LFD15, LHZF16, LWHF16]. The other type of Laplacian matrix is defined from a general real adjacency matrix which need not be nonnegative. This type of Laplacian matrix is called *signed Lapalcian*, and has been found effective in designing cooperative control algorithms to solve formation control and localization in a 3D (and higher-dimensional) space [LWC+16, Zha18, HLZ+17, CWL+17, CLC+16]. For both types – complex and signed Laplacian matrices – their algebraic properties are again essential in characterizing stability and performance of the corresponding cooperative control algorithms. In addition, these algebraic properties are also related to certain connectivity properties of the interaction graph.

The three different types of Laplacian matrices thus offer a new angle to look into the relevant literature on multi-agent cooperative control. Although there are many different cooperative control problems in their appearances, they have a few basic points in common. The interaction graph topology of the agents can be described by graphs, the dynamics of multi-agent systems is hence underlied by Laplacian matrices, and the algebraic properties of these Laplacian matrices dictate stability/performance of the corresponding cooperative control algorithms. These common points therefore allow us to interlink and organize different cooperative control problems and their solutions by different types of Laplacian matrices and the corresponding algebraic properties.

Eight cooperative control problems and their solutions are covered in this book: averaging, optimization, consensus, synchronization, 2D similar formation control, 2D localization, arbitrary-dimensional affine formation control, and arbitrary-dimensional localization. Focus is given exclusively to agents' interaction topology modeled by *directed graphs*. The reason for choosing this focus is multifold. First, directed graphs are more general than undirected graphs; hence the theoretical results of directed graphs include those of undirected graphs as special cases. Second, directed graphs can be more widely applicable, as bidirectional communication may not always possible (e.g. leader-follower structured robotic teams or sensor networks where nodes have heterogeneous communication ranges). Finally, results on directed graphs are scattered in the literature, which calls for an organized presentation. This books serves this purpose.

How to read this book

This book consists of nine chapters:

- Chapter 1: mathematical preliminaries on graphs and their matrices
- Chapters 2–9: eight cooperative control problems

Each chapter is self-contained. Our recommendation is that the reader reads Chapter 1 first, and then feels free to jump to any later chapter on a cooperative control problem of interest. More experienced reader may skip Chapter 1, though we suggest a skim of Sections 1.5 and 1.6 whose content may be less familiar.

Based on different types of Laplacian matrices, Chapters 2–9 are further divided as follows:

- *Standard Laplacian*: Chapters 2–5 (averaging, optimization, consensus, synchronization)
- *Complex Laplacian*: Chapters 6–7 (2D similar formation control, 2D localization)
- *Signed Laplacian*: Chapters 8–9 (arbitrary dimensional affine formation control, arbitrary dimensional localization)

From an alternative angle, Chapters 2–9 are divided into four parts. This division is based on different connectivity conditions on directed graphs.

- *Strongly connected and weight-balanced*: Chapters 2–3 (averaging, optimization)
- *Spanning tree*: Chapters 4–5 (consensus, synchronization)
- *Spanning* 2*-tree*: Chapters 6–7 (2D similar formation control, 2D localization)
- *Spanning multiple tree*: Chapters 8–9 (arbitrary dimensional affine formation control, arbitrary dimensional localization)

Each of these eight chapters is structured similarly. The first two sections provide illustrative examples of the problem studied and explanation of ideas behind the designed algorithm. The third section is technical, with statements of formal results and their proofs (which may be skipped at the first reading). The simulation section presents more illustrative examples of larger-scale networks of agents. The final section provides the main references relevant to the presented algorithms and results. In addition, Chapters 3 and 5 each include an Appendix that introduces background knowledge on the respective subject.

We hope that these different ways of organizing the content of this book provide flexibility to the reader with different purposes. One may choose to read different cooperative control problems independently, or different types of graph Laplacian independently, or graph connectivity conditions progressively.

Who to read this book

This book is written for applied science and engineering students in the graduate level or higher undergraduate levels, as a textbook or a reference for a relevant course. The book is also intended for researchers in systems control, robotics, artificial intelligence, machine learning, signal processing, and computer engineering with interests in multi-agent systems, networked control, and cooperative behaviors.

Where to find additional material

Supplementary material (slides, codes) and updates to this book can be found on the website below:

`https://www.control.eng.osaka-cu.ac.jp/mas`

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