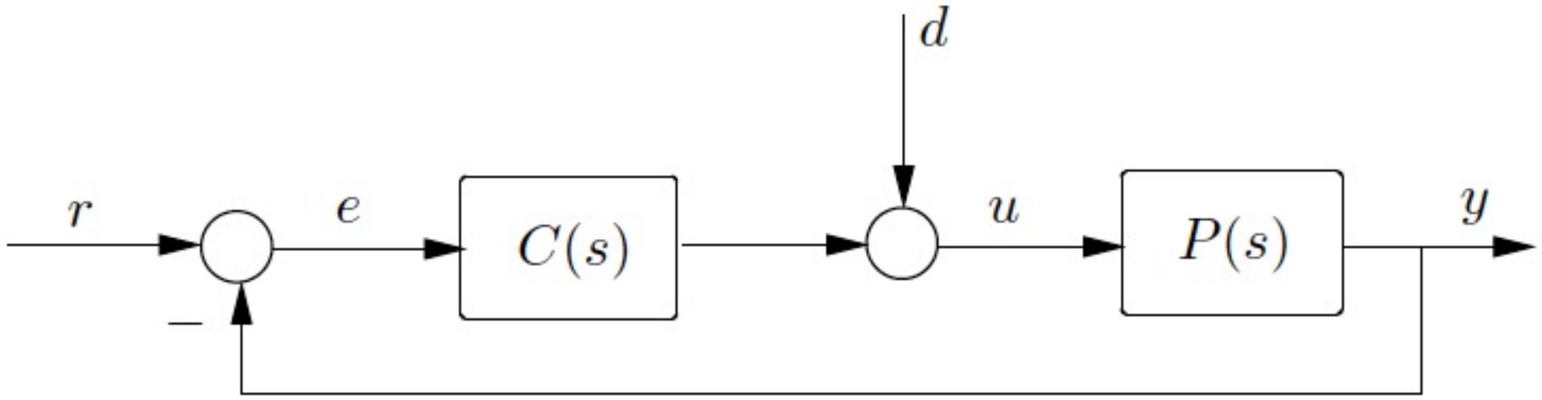
Stability Margin

Distance from instability



Suppose you have determined by Nyquist criterion that a feedback loop is stable; how stable is it?

Namely, how far is it from being unstable?

This can be measured by magnititude and phase of $P(s)C(s)$

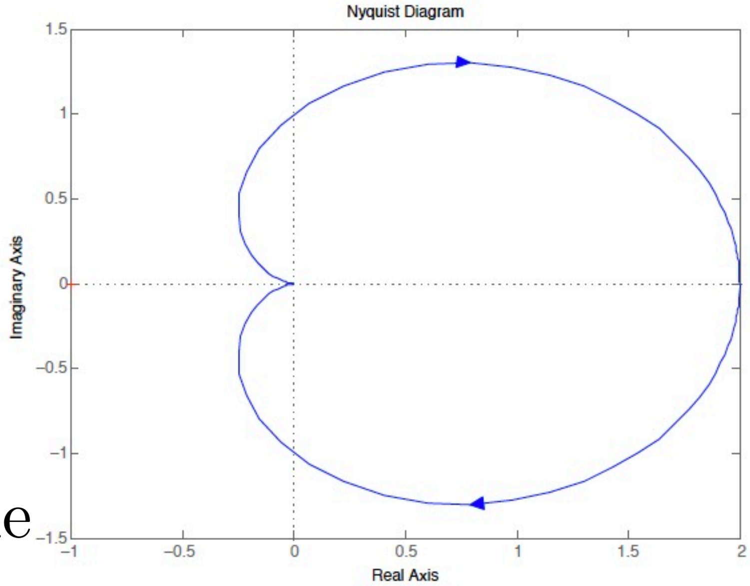
Three measures: phase margin, gain margin, stability margin

Phase margin

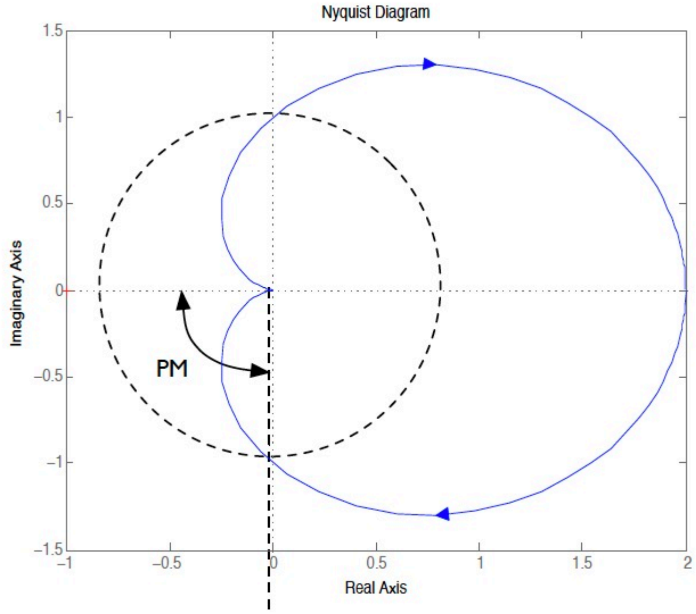
$P(s) = \frac{1}{(s+1)^2}$, $C(s) = 2$, $K = 1$

By Nyquist criterion,
feedback loop is stable

Phase margin: angle from -1
to Nyquist plot crossing unit circle



1) Draw the unit circle
2) Draw the straight radial line from the origin through the point where the unit circle intersects the Nyquist plot
3) PM is the angle from negative real axis to the line drawn in 2)



For this example: PM= 90

Phase margin

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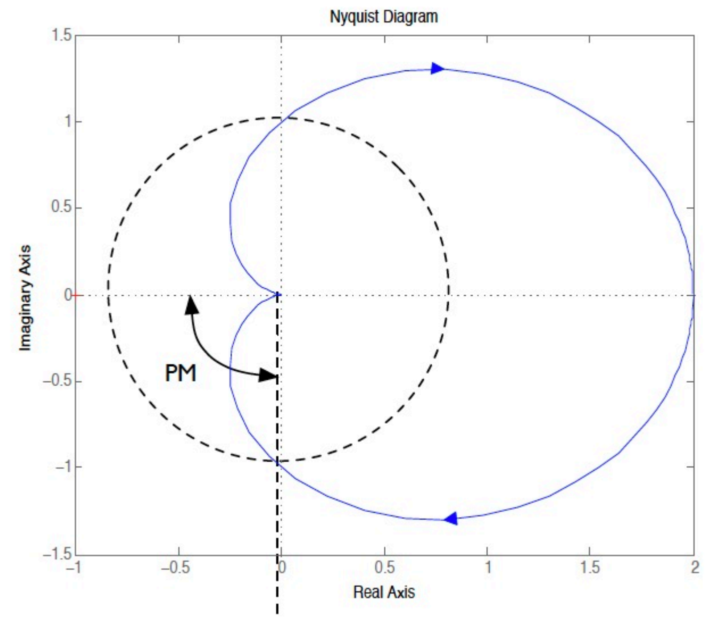

$$\textit{Phase margin}\ \text{PM} = -180 - \angle P(j\omega)C(j\omega),$$
where ω is s.t. $|P(j\omega)C(j\omega)| = 1$

This ω s.t. $|P(j\omega)C(j\omega)| = 1$ is called
gain crossover frequency, denoted ω_{gc}

Phase margin

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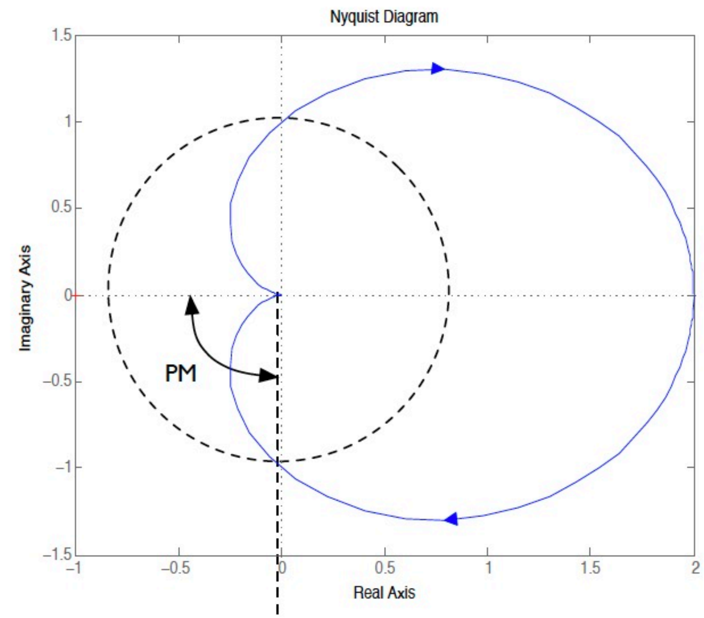


If phase margin is small (say 5 to 10 degrees),
then the closed-loop is close to instability and
there may be "ringing" (oscillatory responses)

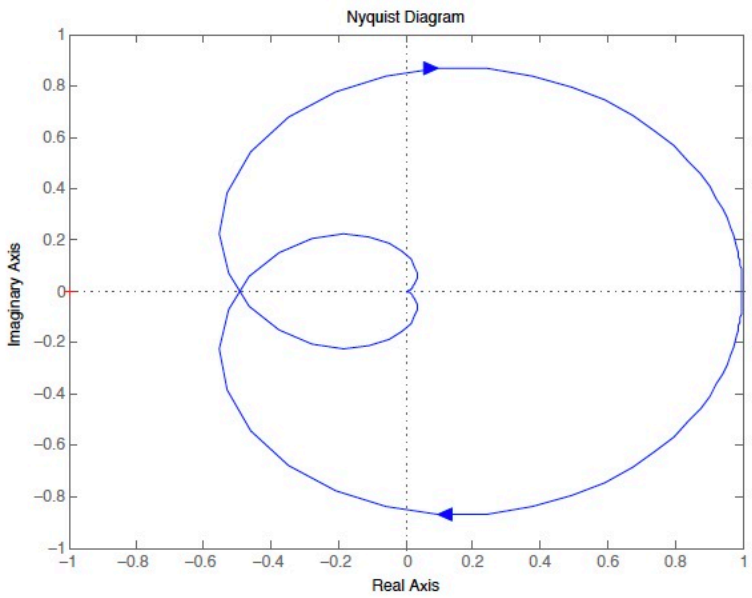
If phase margin is large (say 60 degrees),
the closed-loop can still be close to instability
and hence this measure needs to be used with caution

Gain margin

$P(s)C(s) = \frac{1}{(s+1)^2}\frac{s-1}{s+1}, \; K = 1$

By Nyquist criterion, feedback loop is stable

Gain margin: ratio of -1 and Nyquist plot crossing the negative real axis



1) Find the critical point $-\frac{1}{K}$

2) Find the point where the Nyquist plot intersects the negative real axis

3) GM is the ratio of the two points measured in dB (decible)

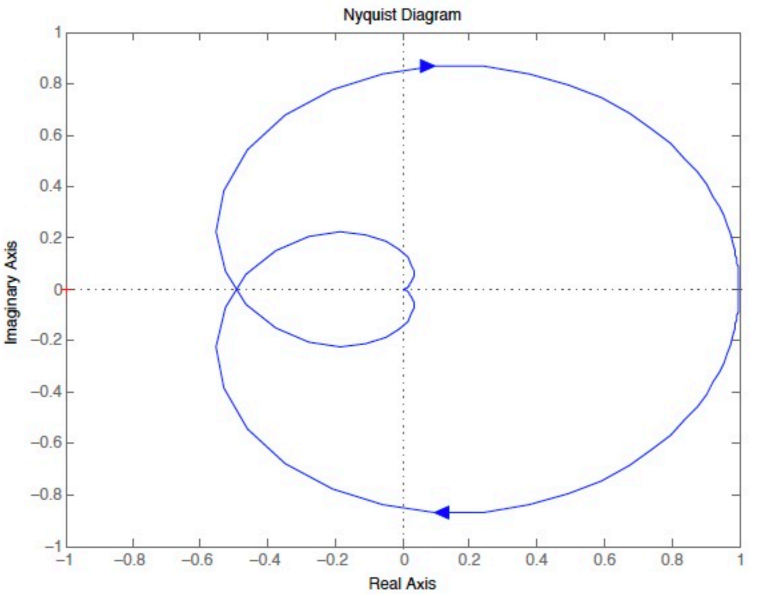
For this example: GM$= 20\log_{10}\frac{-1}{-\frac{1}{2}}$

Gain margin

$P(s)C(s) = \frac{1}{(s+1)^2}\frac{s-1}{s+1},\ K = 1$

By Nyquist criterion, feedback loop is stable

Gain margin: ratio of Nyquist plot crossing the negative real axis with -1



Gain margin GM$= -20\log_{10}|P(j\omega)C(j\omega)|$, where ω is s.t. $\angle P(j\omega)C(j\omega) = -180$

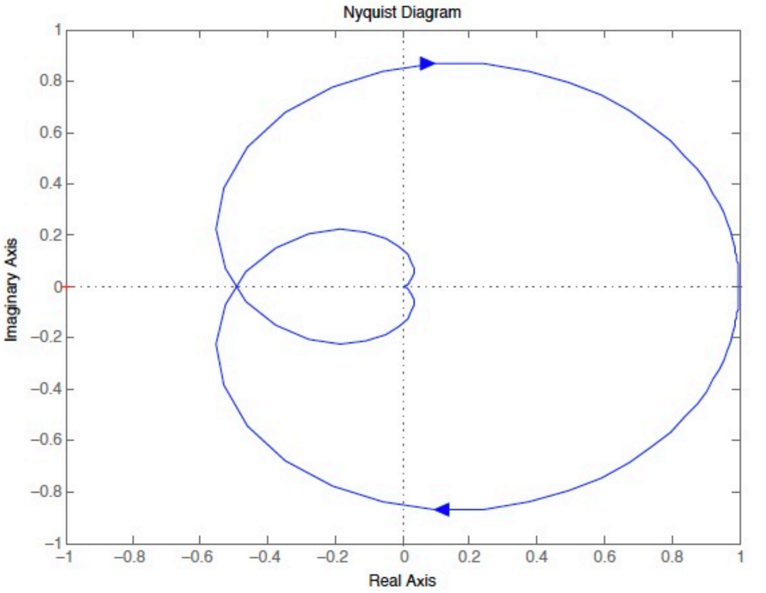
This ω s.t. $\angle P(j\omega)C(j\omega) = -180$ is called *phase crossover frequency*, denoted ω_{pc}

Gain margin

$P(s)C(s) = \frac{1}{(s+1)^2}\frac{s-1}{s+1}, \; K = 1$

By Nyquist criterion,
feedback loop is stable

Gain margin: ratio of
Nyquist plot crossing the
negative real axis with -1



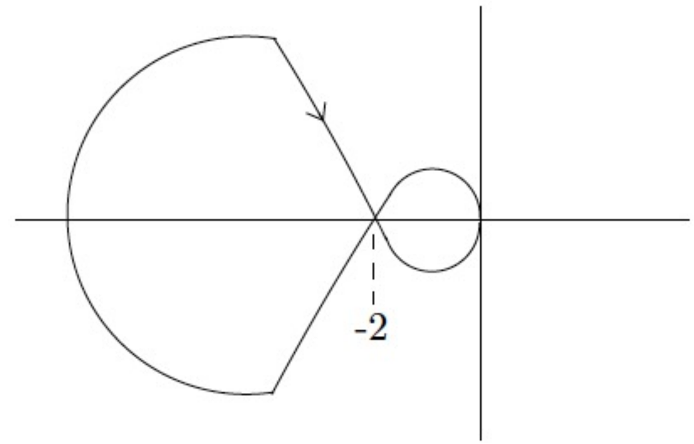
If gain margin is small,
then the closed-loop is close to instability

If gain margin is large,
the closed-loop can still be close to instability
and hence this measure needs to be used with caution

Stability margin

$P(s) = \frac{s+1}{s(s-1)}$, $C(s) = 2$, $K = 1$

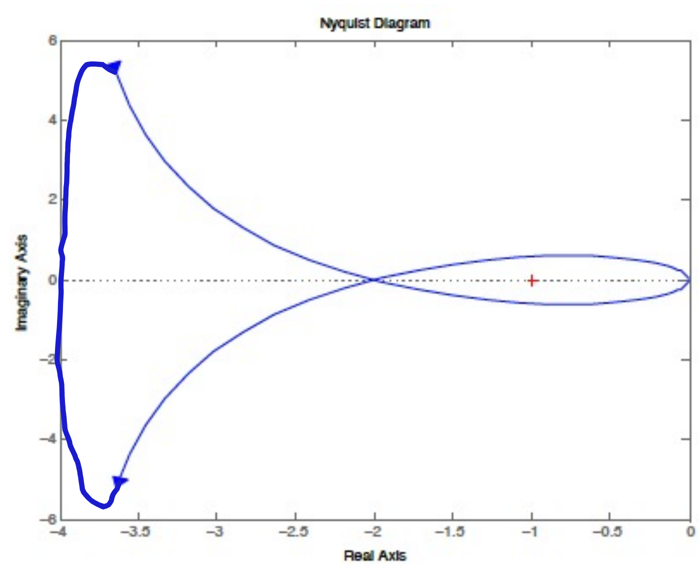
By Nyquist criterion, feedback loop is stable



Stability margin is the distance from the critical point -1 to the closest point on the Nyquist plot

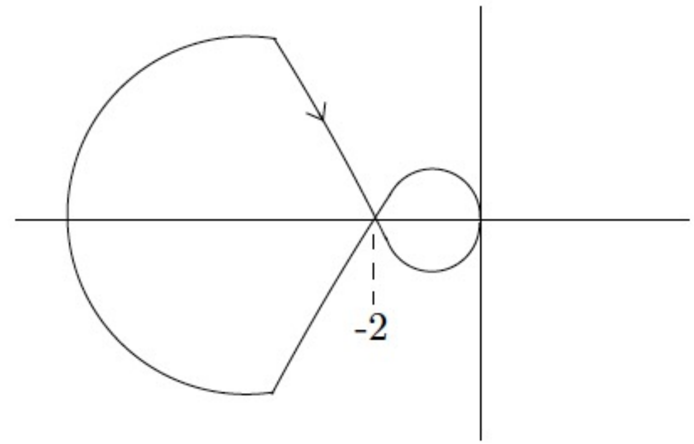
$$\text{SM} = \min_\omega \left| -\frac{1}{K} - P(j\omega)C(j\omega) \right|$$
$$= \min_\omega \left| \frac{1}{K} + P(j\omega)C(j\omega) \right|$$

For this example: SM$= 0.57$



Stability margin

$P(s) = \frac{s+1}{s(s-1)}$, $C(s) = 2$, $K = 1$



By Nyquist criterion,
feedback loop is stable

If stability margin is small,
then the closed-loop is close to instability

If stability margin is large,
then the closed-loop is far from instability