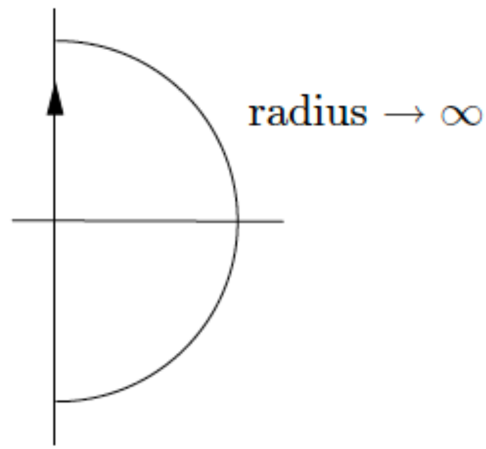
Nyquist Criterion

Principle of the argument

Principle of the argument involves two things:
1) a closed curve $\mathcal{D}$ in the complex plane
2) a transfer function $G(s)$

Nyquist criterion



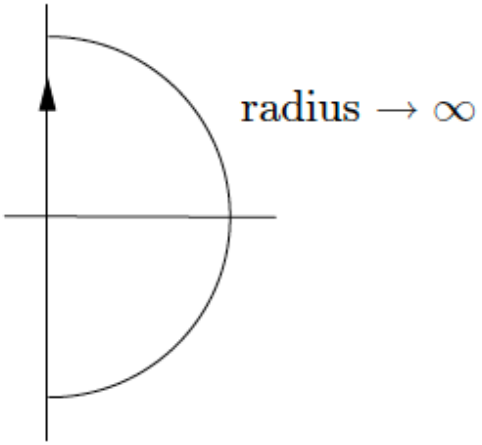
We use a special $\mathcal{D}$, called *Nyquist contour*

Then $\mathcal{G}$ is called *Nyquist plot* of $G(s)$

Suppose that $G(s)$ has no poles or zeros on Nyquist contour $\mathcal{D}$, and has n poles in $\mathrm{Re}(s) > 0$ and m zeros in $\mathrm{Re}(s) > 0$

By principle of the argument, the Nyquist plot $\mathcal{G}$ encircles the origin exactly $n - m$ times CCW

Nyquist criterion

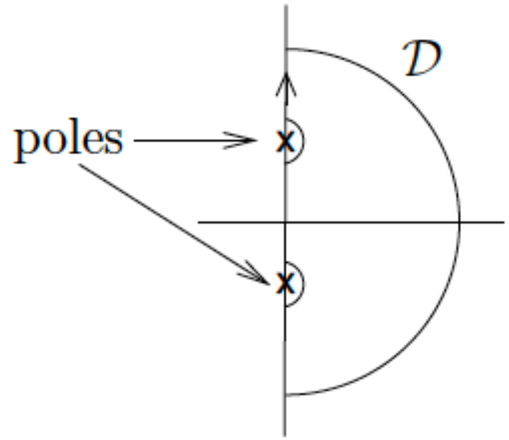


Suppose that $G(s)$ has no poles on Nyquist contour $\mathcal{D}$, and has n poles in $\text{Re}(s) > 0$

$G(s)$ has no zeros in $\text{Re}(s) \geq 0$ if and only if the Nyquist plot $\mathcal{G}$ (i) does not pass through the origin and (ii) encircles the origin exactly n times CCW

This is a graphical test for a rational function $G(s)$ not to have any zeros in the closed right half-plane

Nyquist criterion

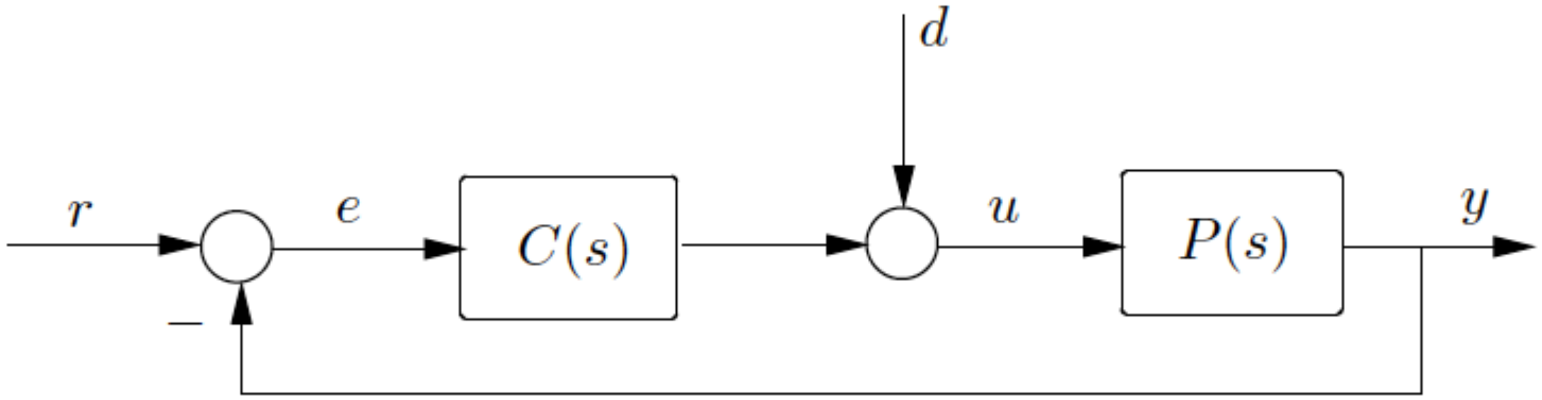


What if $G(s)$ does have poles on Nyquist contour $\mathcal{D}$

Modify $\mathcal{D}$ such that it indents around
these purely imaginary poles

Always indent to the right

Nyquist criterion



Plant $P(s) = \frac{N_p(s)}{D_p(s)}$ is rational and strictly proper,

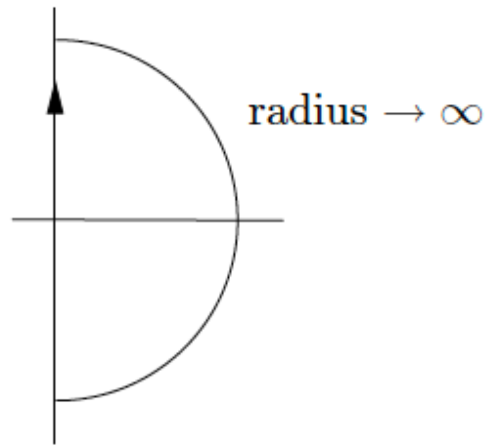
controller $KC(s) = \frac{KN_c(s)}{D_c(s)}$ is rational and proper ($K \neq 0$)

Suppose $P(s)C(s)$ has no pole-zero cancellations in $\mathrm{Re}(s) \geq 0$

Feedback stability $\Leftrightarrow$ $P(s)C(s)$ has no zeros in $\mathrm{Re}(s) \geq -\frac{1}{K}$

Draw the Nyquist plot $\mathcal{G}$ of $G(s) := P(s)C(s)$

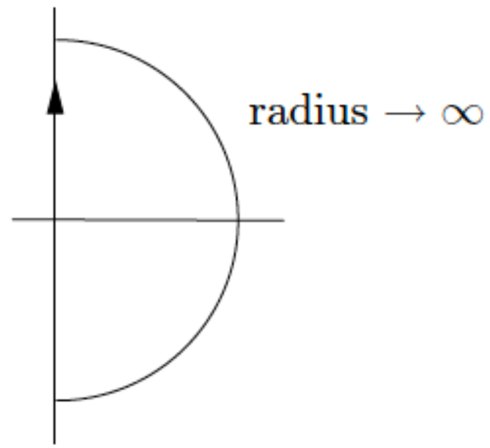
Nyquist criterion



Suppose that $G(s) = P(s)C(s)$ has no poles on the Nyquist contour $\mathcal{D}$ (indenting to the right if necessary), and has n poles in $\mathrm{Re}(s) > 0$

$G(s)$ has no zeros in $\mathrm{Re}(s) \geq -\frac{1}{K}$ if and only if the Nyquist plot $\mathcal{G}$ (i) does not pass through the $-\frac{1}{K}$ and (ii) encircles $-\frac{1}{K}$ exactly n times CCW

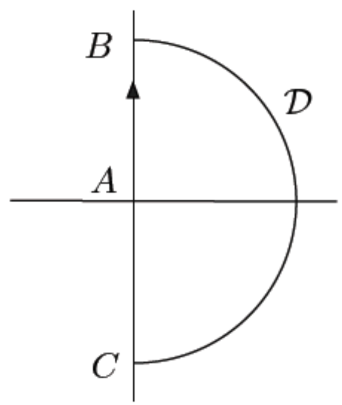
Nyquist criterion

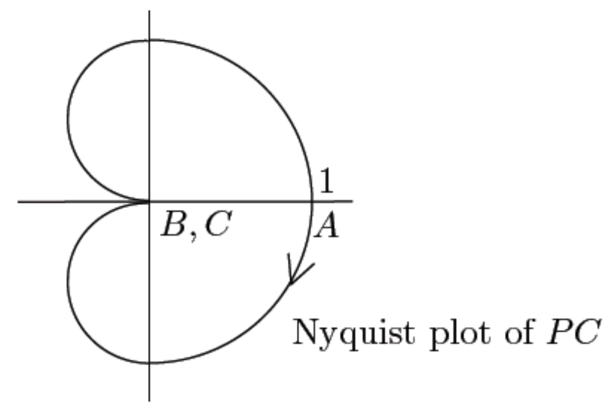


Suppose that $G(s) = P(s)C(s)$ has no poles on the Nyquist contour $\mathcal{D}$ (indenting to the right if necessary), and has n poles in $\mathrm{Re}(s) > 0$

Feedback stability $\Leftrightarrow$
the Nyquist plot $\mathcal{G}$ (i) does not pass through the $-\frac{1}{K}$ and (ii) encircles $-\frac{1}{K}$ exactly n times CCW

Example





$G(s) = P(s)C(s) = \frac{1}{(s+1)^2}$

Divide Nyquist contour $\mathcal{D}$ into 3 segments:

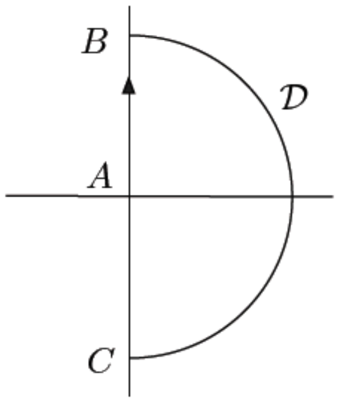
Segment from A to B: $s = j\omega$, ω from 0 to ∞

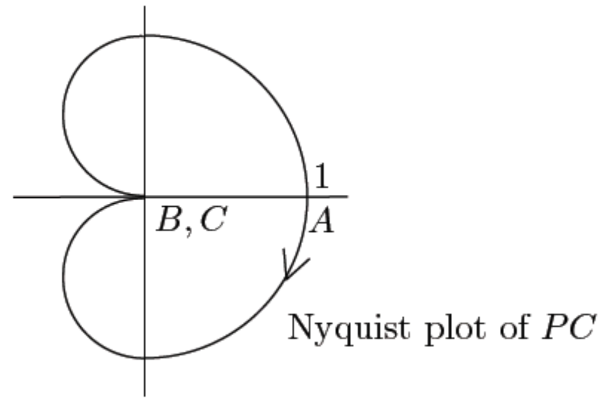
$P(j\omega)C(j\omega) = \frac{1}{(j\omega+1)^2}$

$P(0)C(0) = 1$ $\qquad$ $P(j\infty)C(j\infty) \to \frac{1}{(j\omega)^2} = -\frac{1}{\omega^2}$

$\mathrm{Re}(P(j\omega)C(j\omega)) = \frac{1-\omega^2}{(1-\omega^2)^2+(2\omega)^2}$, $\mathrm{Im}(P(j\omega)C(j\omega)) = \frac{-2\omega}{(1-\omega^2)^2+(2\omega)^2}$

Example





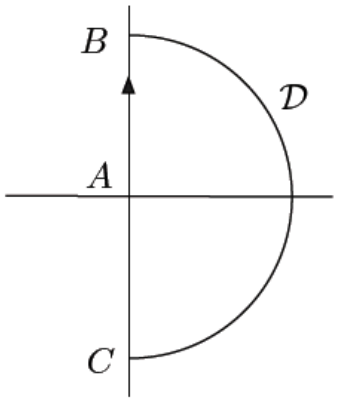
$G(s) = P(s)C(s) = \frac{1}{(s+1)^2}$

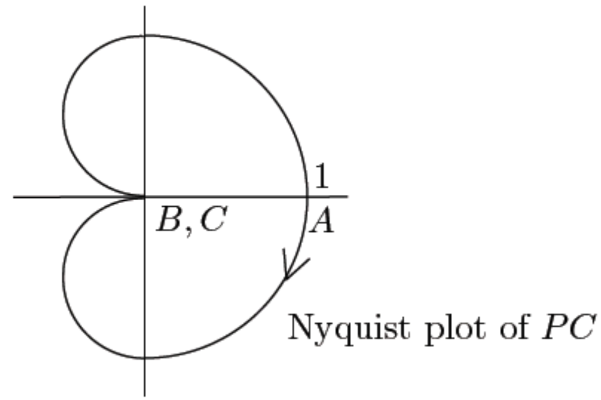
Divide Nyquist contour $\mathcal{D}$ into 3 segments:

Segment from B to C: radius is ∞

$P(j\infty)C(j\infty) = 0$, $P(-j\infty)C(-j\infty) = 0$

Example





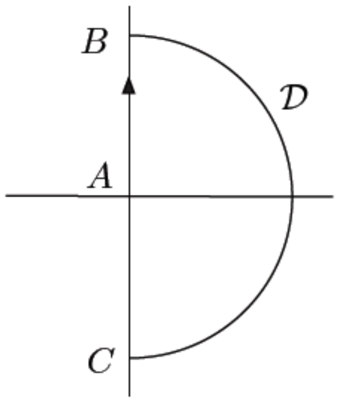
$G(s) = P(s)C(s) = \frac{1}{(s+1)^2}$

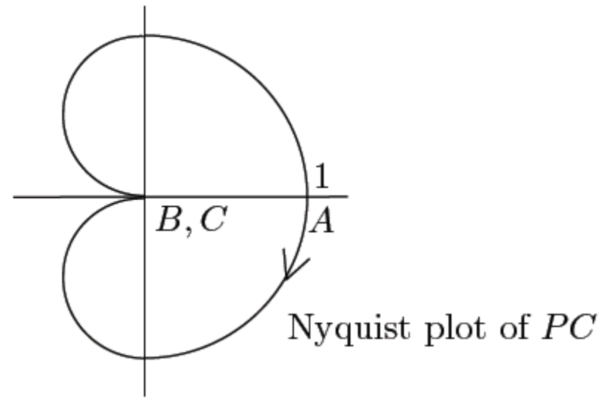
Divide Nyquist contour $\mathcal{D}$ into 3 segments:

Segment from C to A: $s = j\omega$, ω from $-\infty$ to 0

complex conjugate of the segment from A to B

Example





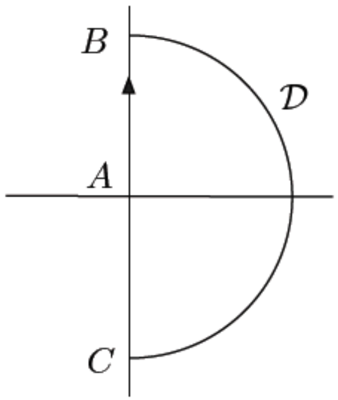
$G(s) = P(s)C(s) = \frac{1}{(s+1)^2}$
$P(s)C(s)$ has no poles in $\mathrm{Re}(s) > 0$, i.e. $n = 0$

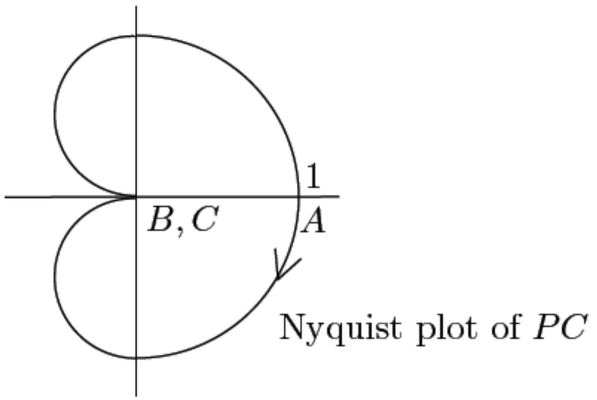
By Nyquist criterion: feedback loop is stable iff the Nyquist plot $\mathcal{G}$ (i) does not pass through $-\frac{1}{K}$ and (ii) encircles $-\frac{1}{K}$ exactly 0 time CCW

So either $-\frac{1}{K} < 0$ or $-\frac{1}{K} > 1$, i.e. either $K > 0$ or $-1 < K < 0$

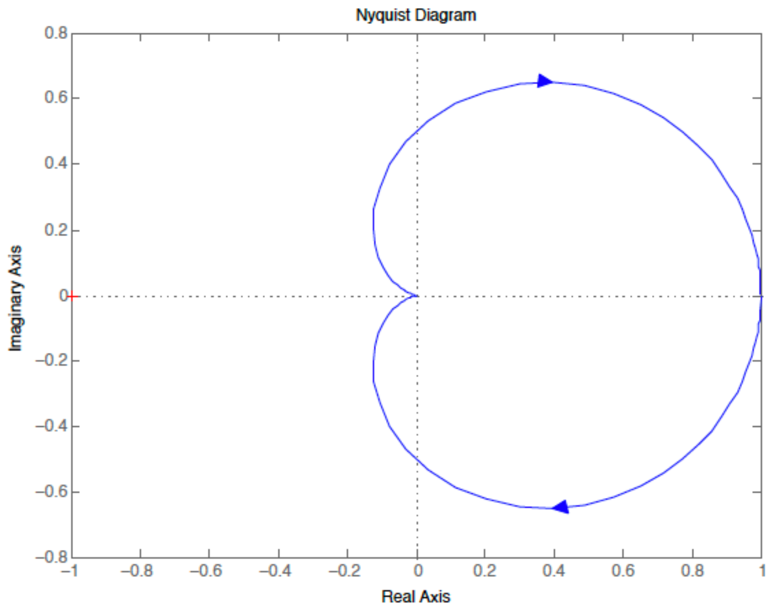
But $K = 0$ is fine (why?); so $K > -1$ after all

Example





Nyquist plot is not accurate




```
s=tf('s');
G=1/(s+1)^2;
nyquist(G)
```