THEORY OF COMPUTATION

Homework 4 — 20% of your final score

Assigned 2020.12.22

Deadline 2021.01.19 (ZERO point for late submission without valid reasons)

Name your document "YourStudentNumber-YourSurname-HW4".

Write all necessary steps for your answers. Just writing the final answers is not acceptable.

Your answers can be English or Japanese.

Five (5) points for each problem

1. Consider the following (non-context-free) language

$$L = \{\alpha^n \beta^n \gamma^n \mid n = 0, 1, 2, \ldots\}.$$

(1.1) Design a Turing machine **M** to accept/recognize L. Draw its state transition diagram with tape operations.

(1.2) For the Turing machine **M** you designed in (1.1), specify its 7 components Q, Σ, Γ, δ, q_0, q_a, q_r.

(1.3) Is the language L Turing-decidable? (Explain your answer.)

2. Let the alphabet be $\Sigma = \{0, 1\}$ and consider the following high-level Turing machine **M**:

M = "On an input string $s \in \Sigma^*$:

(i) Scan the tape and mark the first 0 which has not been marked. If no unmarked 0 is found, go to step (iii). Otherwise, move the tape head back to the leftmost of the tape.

(ii) Scan the tape and mark the first 1 which has not been marked. If no unmarked 1 is found, *reject* s. Otherwise, move the tape head back to the leftmost of the tape, and go back to step (i).

(iii) Move the tape head back to the leftmost of the tape. Scan the tape to check if there is any more unmarked 1 left. If there is no more unmarked 1, *accept* s; otherwise, *reject* s."

What is the language L that this Turing machine accepts/recognizes?

3. Consider the following problem: given two DFA $\mathbf{G}_1$ and $\mathbf{G}_2$, test if they accept the same language, i.e. $L_a(\mathbf{G}_1) = L_a(\mathbf{G}_2)$.

Prove that this problem is algorithmically solvable. HINT: first write down the language L that corresponds to the above problem; then design a high-level Turing machine (i.e. algorithm) that *decides* the language L.

4. Let L_1 and L_2 be two Turing-decidable languages.

(4.1) Prove that their union $L_1 \cup L_2$ is also Turing-decidable.

(4.2) Prove that their intersection $L_1 \cap L_2$ is also Turing-decidable.

(4.3) Prove that their catenation $L_1 L_2$ is also Turing-decidable.
