THEORY OF COMPUTATION

Homework 5 — 20% of your final score

Assigned 2021.01.19

Deadline 2021.02.02 (ZERO point for late submission without valid reasons)

Name your document "YourStudentNumber-YourSurname-HW5".

Write all necessary steps for your answers. Just writing the final answers is not acceptable.

Your answers can be English or Japanese.

Five (5) points for each problem

1. Let the alphabet be $\Sigma = \{0, 1\}$, and consider the language

$$L = \{\#s_1\#s_2\#s_3 \mid s_1, s_2, s_3 \in \Sigma^* \text{ and } s_1 = s_2 \text{ and } s_2 \neq s_3\}.$$

Either prove that L is decidable, by designing a (high-level) Turing machine that decides L; or prove that L is undecidable.

2. Answer the following:

(2.1) If $f(n) = 2n^4 + 2^{50}n^2 + 10n$, then $f(n) = O(?)$

(2.2) If $f(n) = 9n \log_2 n + 4n \log_2 \log_2 n + 6$, then $f(n) = O(?)$

(2.3) If $f(n) = 9n^4 + n2^n$, then $f(n) = O(?)$

(2.4) If $f(n) = O(n^3) + O(n^2 \log n) + O(1)$, then $f(n) = O(?)$

(2.5) If $f(n) = O(n^2(\log n)^2) + O(n^2 \log(\log n))$, then $f(n) = O(?)$

3. Consider the Turing machine (i.e. algorithm) you designed for Problem 3 in Homework 4, and suppose that the numbers of states of $\mathbf{G}_1$ and $\mathbf{G}_2$ are no more than n.

(3.1) Analyze the (worst-case) time complexity of the algorithm using the big-O notation with respect to n.

(3.2) Let L be the language that your designed algorithm decides. Is L in class P or class NP? (Explain your answer.)

4. Consider an undirected graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, where $\mathcal{V}$ is the set of nodes and $\mathcal{E}$ is the set of edges. Let $v_1, v_2 \in \mathcal{V}$ be two (arbitrary) nodes and $k \geq 1$ be a positive integer.

Problem: determine if there is a path (i.e. a sequence of edges) from v_1 to v_2, and the path length (i.e. the number of edges) is less than or equal to k.

(4.1) Write down the language L corresponding to the above problem.

(4.2) Design an algorithm (i.e. Turing machine) that decides L.

(4.3) Let the number of nodes in $\mathcal{V}$ be n (i.e. $|\mathcal{V}| = n$). Analyze the time complexity of your designed algorithm using the big-O notation with respect to n. Then conclude if the language L is in class P or class NP.
