# Configurations of Turing Machines

# Turing machine

A (deterministic) Turing machine (TM) $\mathbf{M}$ is a 7-tuple $\mathbf{M} = (Q, \Sigma, \Gamma, \delta, q_0, q_a, q_r)$, where

$Q$ : state set; a finite set of states

$\Sigma$ : alphabet; a finite set of symbols; $\mathrm{B} \notin \Sigma$

$\Gamma$ : tape alphabet; a finite set of symbols used in the tape; $\mathrm{B} \in \Gamma$ and $\Sigma \subseteq \Gamma$

$\delta : Q \times \Gamma \to Q \times \Gamma \times \{\mathrm{L}, \mathrm{R}\}$: transition function

$q_0 \in Q$ : initial state

$q_a \in Q$ : accept state

$q_r \in Q$ : reject state $(q_r \neq q_a)$

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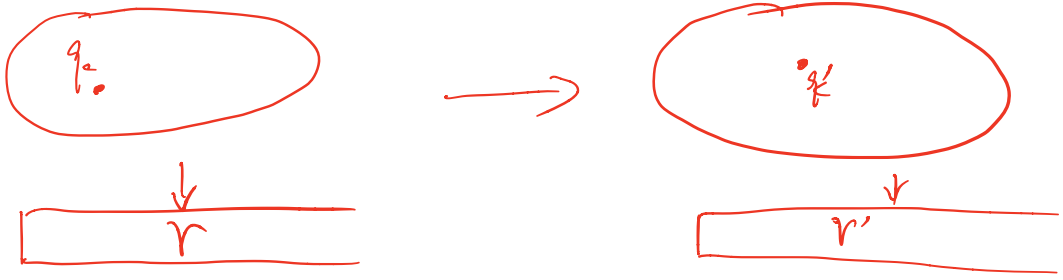
$\delta(q, \gamma) = (q', \gamma', \mathrm{R})$:
before transition: state $q$, tape head $\gamma$
after transition: state $q'$, overwrite by $\gamma'$, head moves right



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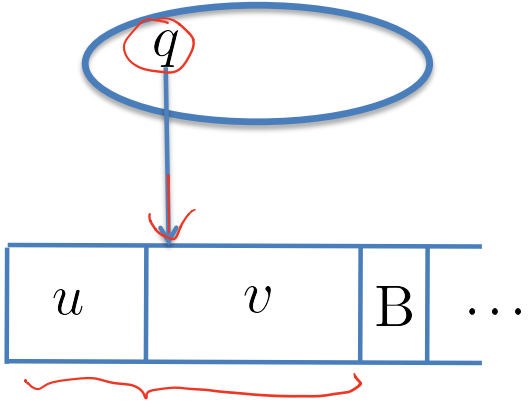
Let $q \in Q$ and $u, v \in \Gamma^*$.
Then $uqv$ is a configuration:
state $q$, tape content $uv$, head at first symbol of $v$

TM is at "$uqv$" configuration.



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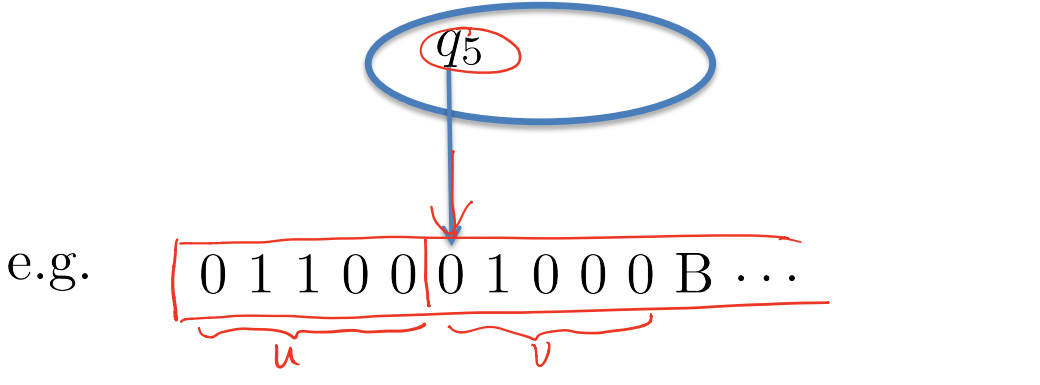

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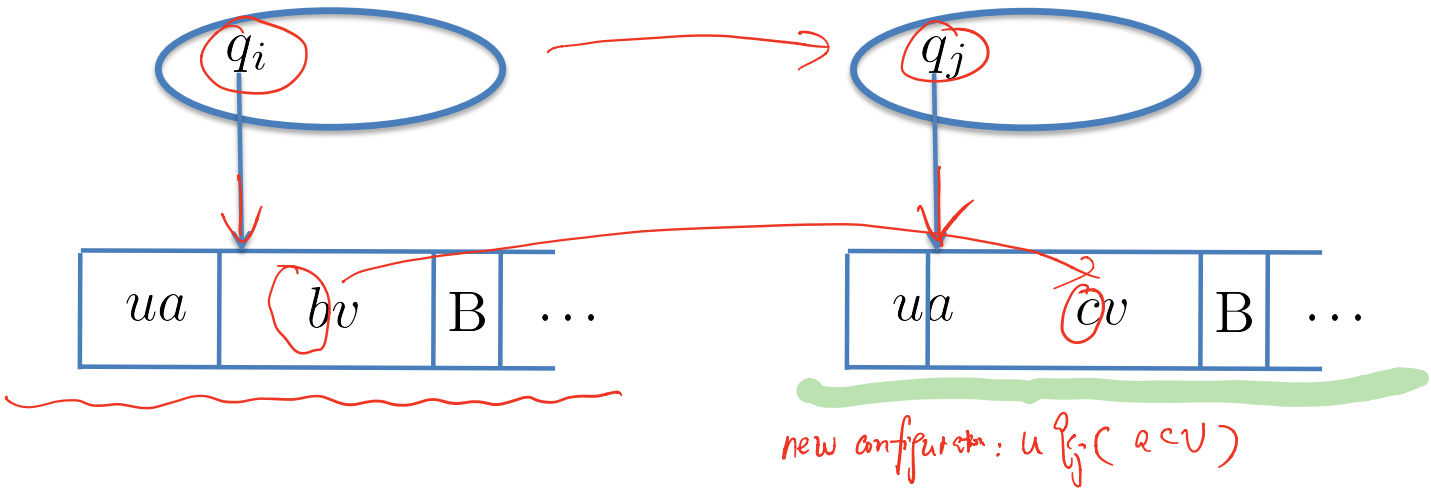
Let $q_i, q_j \in Q$, $a, b, c \in \Gamma$, and $u, v \in \Gamma^*$.
Consider configuration $uaq_ibv$ and transition $\delta(q_i, b) = (q_j, c, L)$.
Then the resulting configuration is $uq_jacv$



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Say configuration $uaq_ibv$ *derives* configuration $uq_jacv$

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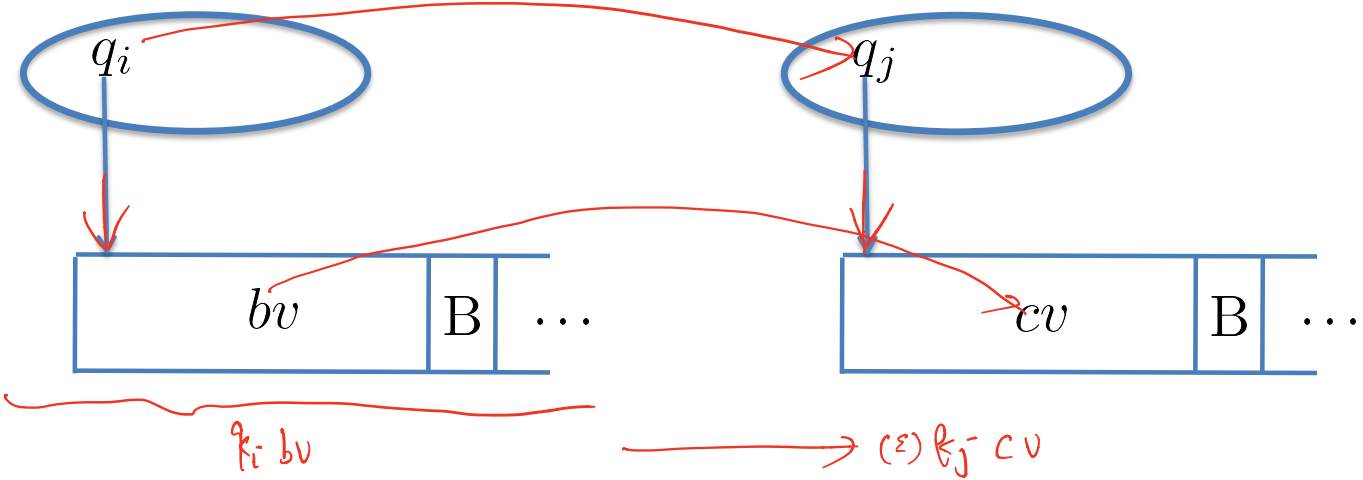
Then the derived configuration is $q_j cv$



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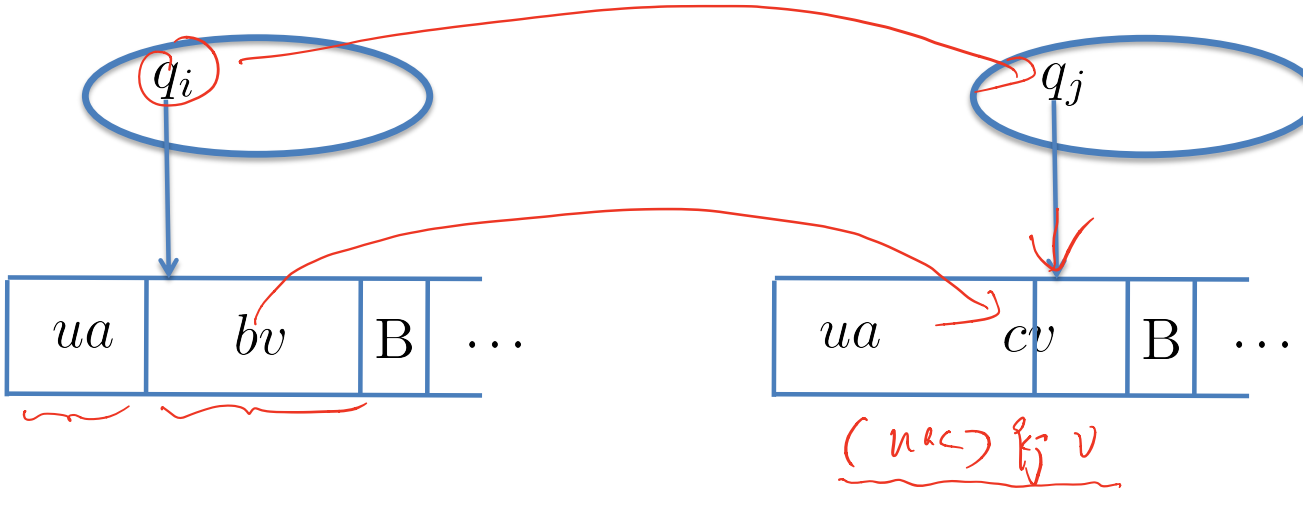
Then the derived configuration is $uacq_jv$



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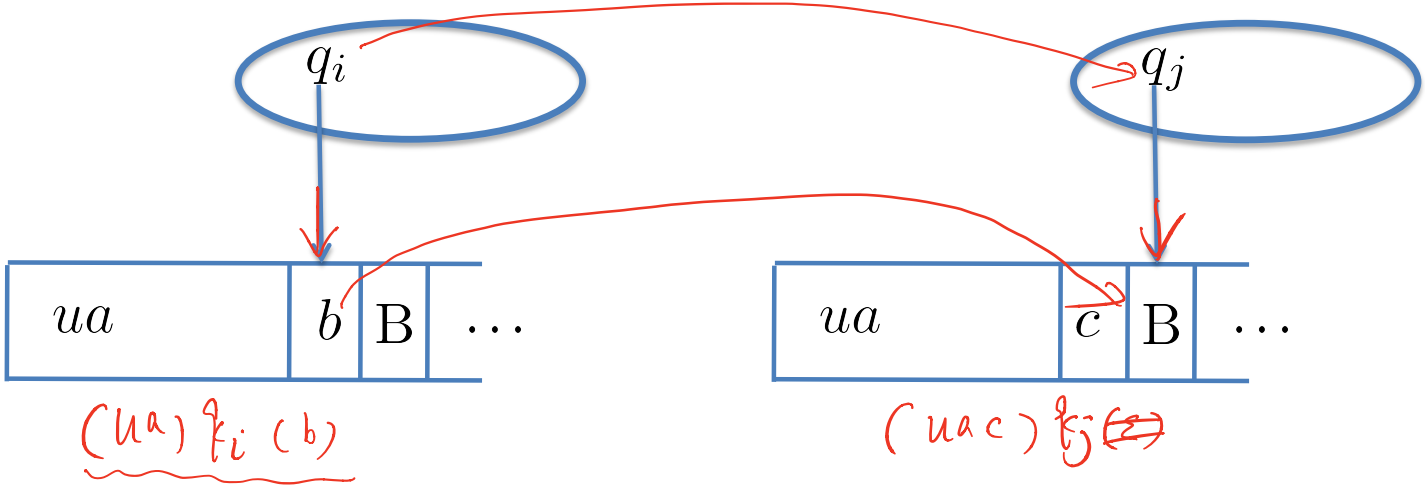
Consider configuration $uaq_ib$ and transition $\delta(q_i, b) = (q_j, c, R)$.
Then the derived configuration is $uacq_j$



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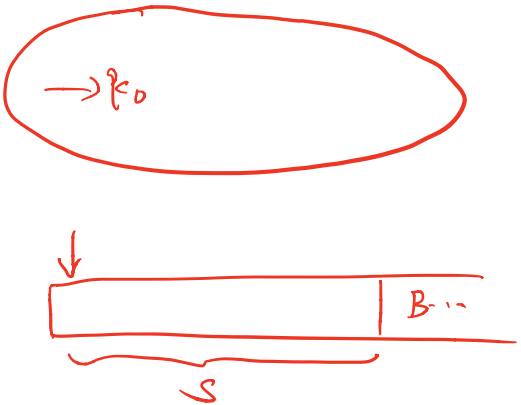
Three special configurations:
Start configuration: $q_0 s$, where $s$ is the input string
Accept configuration: $uq_a v$
Reject configuration: $uq_r v$



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Start configuration: $q_0 s$, where $s$ is the input string
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Reject configuration: $uq_r v$
(halting configurations)

The (configuration) transition of TM can halt
by reaching a halting configuration.

If this never happens, TM loops forever.