LINEAR FEEDBACK CONTROL - HOMEWORK 2

Assigned 2018.11.05. Submission deadline 2018.11.19 (for only those who want their homework to be marked).

Problems

1. For $\dot{x} = Ax$ we defined that the origin $x = 0$ is stable (in the sense of Lyapunov) if

$$(\forall \epsilon > 0)(\exists \delta > 0)(\forall x(0) \in \mathbb{R}^n)||x(0)|| < \delta \Rightarrow (\forall t \geq 0)||x(t)|| < \epsilon$$

By negating the above logical expression, derive the definition that the origin $x = 0$ is *unstable*.

2. Let

$$A = \begin{bmatrix} a & b \\ -b & a \end{bmatrix}$$

Compute e^{At}.

For the linear system $\dot{x} = Ax$, is the origin $x = 0$ stable, asymptotically stable, or unstable? Explain why.

3. Let $A_1, A_2 \in \mathbb{R}^{n \times n}$ and $t \geq 0$. Prove that $e^{A_1 t} e^{A_2 t} = e^{(A_1 + A_2)t}$ if and only if $A_1 A_2 = A_2 A_1$ (i.e. A_1 and A_2 commute).

4. Consider the cart-pendulum system in HOMEWORK 1 and the linearized system $\dot{x} = Ax$. Is the origin $x = 0$ stable, asymptotically stable, or unstable? Explain why.

Bonus problem (Matlab)

In this homework, we introduce how to compute vector norms; matrix eigenvalues and eigenvectors. Given a vector v and a matrix A as follows:

$$v = \begin{bmatrix} 1 \\ 0.5 \\ -2 \end{bmatrix}, \quad A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

the 2-norm of v can be computed by

n_v =norm(v)

The 1-norm and infinite-norm of v can also be computed by this function. Please check out by typing

help norm

To compute the eigenvalues and eigenvectors of A use

[V D] =eig(A)

where V's columns are eigenvectors, and D's diagonal entries are eigenvalues. For a non-diagonalizable matrix A, use instead

[V D] =jordan(A)