LINEAR FEEDBACK CONTROL - HOMEWORK 3

Assigned 2018.11.19. Submission deadline 2018.12.03 (for only those who want their homework to be marked).

Problems

1. Let $\lambda \in \mathbb{C}$ be a complex number and consider

$$\begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix}, \quad \begin{bmatrix} \lambda & 1 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix}, \quad \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 1 \\ 0 & 0 & \lambda \end{bmatrix}, \quad \begin{bmatrix} \lambda & 1 & 0 \\ 0 & \lambda & 1 \\ 0 & 0 & \lambda \end{bmatrix}$$

The above matrices all have eigenvalues λ (and only λ). Which of the above matrices are Jordan blocks for the eigenvalue λ?

Let

$$A = \begin{bmatrix} \lambda & 1 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix}$$

Compute e^{At}.

For the linear system $\dot{x} = Ax$, is the origin $x = 0$ stable, asymptotically stable, or unstable? Explain why.

2. Consider the cart-pendulum system in HOMEWORK 1 and the linearized system $\dot{x} = Ax + Bu$. Is the pair (A, B) controllable? Explain why.

3. For the LTI system $\dot{x} = Ax + Bu$ and $y = Cx$ with initial state $x(0) = x_0$, we have shown that the relation between output y and input u is as follows:

$$y(t) = Ce^{At}x_0 + C\int_0^t e^{A(t-\tau)}Bu(\tau)d\tau$$

Prove that the system is indeed "linear", i.e. if u_1 produces y_1 and u_2 produces y_2, then $c_1u_1 + c_2u_2$ produces $c_1y_1 + c_2y_2$ for all real numbers $c_1, c_2 \in \mathbb{R}$.

Bonus problem (Matlab)

In this homework, we introduce how to use Matlab to simulate an LTI system $\dot{x} = Ax$ given an initial condition $x(0)$. Create a matrix A as follows:

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

The following commands simulate the trajectory of $x(t)$ starting from $x(0) = [-3\ 1\ 4.3]^\top$:

```
t=0:0.01:10;
x0 = [-3; 1; 4.3];
dxdt = @(t,x)(A)*x;
[T,x]= ode45(dxdt,t,x0);
figure
hold on
plot(t,x(:,1),'r')
plot(t,x(:,2),'b')
plot(t,x(:,3),'y')
```

Please check by yourself the meaning of the above commands.

In addition we introduce how to use symbolic variables and compute matrix exponentials. Create a matrix A as follows:

$$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$$

The following command creates a symbolic variable:

```
syms t
```

Then use the following command to compute e^{At}:

```
expm(A*t)
```
