LINEAR FEEDBACK CONTROL - HOMEWORK 1

Assigned 2019.10.08. Submission deadline 2019.10.29 (for only those who want their homework to be marked).

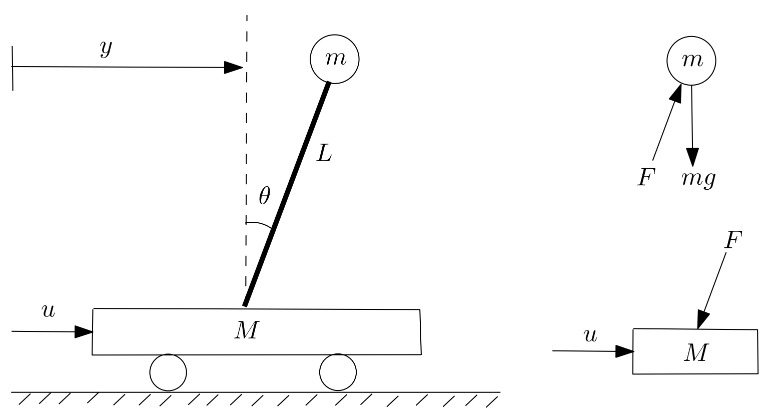
Problems

1. Consider the following ordinary differential equation (ODE)

$$\ddot{y} - 2y = 3\dot{u}$$

Derive the state model (in the form of $\dot{x} = Ax + Bu$ and $y = Cx$).

2. Consider a cart-pendulum system



θ: angle of the pendulum deviated from the upright direction

L: length of the pendulum

m: mass of the ball

M: mass of the cart

g: gravitational acceleration

y: position of the cart

u: force applied to the cart

F: force along the pendulum

For the ball, in the horizontal direction

$$F \sin\theta = m\frac{d^2}{dt^2}(y + L\sin\theta) \tag{1}$$

in the vertical direction

$$mg - F\cos\theta = m\frac{d^2}{dt^2}(L - L\cos\theta) \tag{2}$$

For the cart,

$$u - F\sin\theta = M\frac{d^2 y}{dt^2} \tag{3}$$

Choose the state variable $x = [x_1\ x_2\ x_3\ x_4] = [y\ \dot{y}\ \theta\ \dot{\theta}]^\top$.

2.1. Suppose that the angle θ is very small (i.e. $\sin\theta \approx \theta$ and $\cos\theta \approx 1$). From (1), (2), (3) derive the state model (in the form of $\dot{x} = Ax + Bu$ and $y = Cx$).

2.2. From (1), (2), (3) derive the nonlinear model (check the details for yourself)

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} x_2 \\ \frac{u + mLx_4^2\sin x_3 - mg\sin x_3\cos x_3}{M + m\sin^2 x_3} \\ x_4 \\ \frac{-u\cos x_3 - mLx_4^2\sin x_3\cos x_3 + (m+M)g\sin x_3}{L(M + m\sin^2 x_3)} \end{bmatrix}$$

Linearize the above model (by computing Jacobians) in the neighborhood of the equilibrium point $x^* = [1\ 0\ 0\ 0]$ and $u^* = 0$ to derive $\dot{\Delta x} = A\Delta x + B\Delta u$. Check if matrices A, B are the same as in 2.1.

Bonus problem (Matlab)

Matlab is a software useful for control systems modeling and design. Our university has a license of Matlab 2015b. You can download the install files from the course website https://www.control.eng.osaka-cu.ac.jp/teaching/linear2019.

There are many tutorials on the basics of Matlab. One is at https://matlabacademy.mathworks.com/jp.

In this homework, we introduce how to create state models and transfer functions, as well as transform from one to the other. Given

$$A = \begin{bmatrix} 0 & 1 \\ 0 & -1 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ \frac{1}{2} \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 \end{bmatrix}, \quad D = 0$$

to create the state model with these matrices, execute

ss1 =ss(A, B, C, D)

The next line

$$\text{tf1 =tf}([0.5],[1\ 1\ 0])$$

creates a transfer function

$$G(s) = \frac{0.5}{s^2 + s}$$

To convert tf1 to a state model, execute

$$\text{ss2 = ss(tf1)}$$

Finally to convert ss1 to a transfer line, execute

$$\text{tf2 = tf(ss1)}$$

Install Matlab 2015b and try the above procedures.
