

## LINEAR FEEDBACK CONTROL - HOMEWORK 2

Assigned 2019.11.05. Submission deadline 2019.11.19 (for only those who want their homework to be marked).

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### Problems

1. For  $\dot{x} = Ax$  we defined that the origin  $x = 0$  is stable (in the sense of Lyapunov) if

$$(\forall \epsilon > 0)(\exists \delta > 0)(\forall x(0) \in \mathbb{R}^n) \|x(0)\| < \delta \Rightarrow (\forall t \geq 0) \|x(t)\| < \epsilon$$

By negating the above logical expression, derive the definition that the origin  $x = 0$  is *unstable*.

2. Let

$$A = \begin{bmatrix} a & b \\ -b & a \end{bmatrix}$$

Compute  $e^{At}$ .

For the linear system  $\dot{x} = Ax$ , is the origin  $x = 0$  stable, asymptotically stable, or unstable? Explain why.

3. Let  $A_1, A_2 \in \mathbb{R}^{n \times n}$  and  $t \geq 0$ . Prove that  $e^{A_1 t} e^{A_2 t} = e^{(A_1 + A_2)t}$  if and only if  $A_1 A_2 = A_2 A_1$  (i.e.  $A_1$  and  $A_2$  commute).

4. Consider the cart-pendulum system in HOMEWORK 1 and the linearized system  $\dot{x} = Ax$ . Is the origin  $x = 0$  stable, asymptotically stable, or unstable? Explain why.

### Bonus problem (Matlab)

In this homework, we introduce how to compute vector norms; matrix eigenvalues and eigenvectors. Given a vector  $v$  and a matrix  $A$  as follows:

$$v = \begin{bmatrix} 1 \\ 0.5 \\ -2 \end{bmatrix}, \quad A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

the 2-norm of  $v$  can be computed by

$$\text{n\_v} = \text{norm}(v)$$

The 1-norm and infinite-norm of  $v$  can also be computed by this function. Please check out by typing

`help norm`

To compute the eigenvalues and eigenvectors of  $A$  use

`[V D] = eig(A)`

where  $V$ 's columns are eigenvectors, and  $D$ 's diagonal entries are eigenvalues. For a non-diagonalizable matrix  $A$ , use instead

`[V D] = jordan(A)`

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