LINEAR FEEDBACK CONTROL - HOMEWORK 4

Assigned 2019.12.03. Submission deadline 2020.01.07 (for only those who want their homework to be marked).

Problems

1. Consider $\dot{x} = Ax + Bu$, where

$$A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

The eigenvalues of A are j and $-j$ (here $j := \sqrt{-1}$); thus the origin is stable but not asymptotically stable.

(1.1) Is (A, B) controllable? (Explain your answer.)

(1.2) Consider state feedback $u = Fx$, and suppose that the desired eigenvalues of $A + BF$ are $-1, -2$ (so that the closed-loop system is asymptotically stable). Compute F.

2. Consider the cart-pendulum system in HOMEWORK 1 and the linearized system $\dot{x} = Ax+Bu$. You have checked in HOMEWORK3 that (A, B) is controllable. Now for simplicity let $m = M = L = 1$ and consider state feedback $u = Fx$. Suppose that the desired eigenvalues of the closed-loop matrix $A + BF$ are $-1, -2, -1 + j, -1 - j$ (so that the closed-loop system is asymptotically stable). Compute F.

3. Consider a double inverted-pendula system: $\dot{x} = Ax + Bu$ where

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{g}{l} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & \frac{g}{L} & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ \frac{1}{ml} \\ 0 \\ \frac{1}{ML} \end{bmatrix}.$$

Here m, M are the masses of the two pendula; l, L are the lengths of the two pendula; and g is the gravitational acceleration.

(3.1) Is (A, B) is controllable? (Explain your answer.)

(3.2) If (A, B) is not controllable, decompose this (uncontrollable) system into a controllable subsystem and an uncontrollable subsystem.

(3.3) Is (A, B) stabilizable? If so, design state feedback $u = Fx$ to stabilize the system.

Bonus problem (Matlab)

In this homework, we introduce a few convenient Matlab functions.

(i) Let A be a matrix. rank(A) computes the rank of A, and poly(A) computes the characteristic polynomial of A.

(ii) Let A, B be matrices of suitable sizes. ctrb(A, B) computes the controllability matrix of the pair (A, B).

(iii) Let A, B, C be matrices of suitable sizes. [$\tilde{A}$,$\tilde{B}$,$\tilde{C}$,V]=ctrbf(A, B, C) decomposes the system given by (A, B, C) into controllable and uncontrollable parts given by $(\tilde{A}, \tilde{B}, \tilde{C})$, and V is the invertible matrix for the corresponding similarity transformation.

(iv) Let A, B be matrices of suitable sizes, and $Eigs$ be an array of desired closed-loop eigenvalues. K=place($A, B, Eigs$) or K=acker($A, B, Eigs$) computes the gain matrix K for state feedback $u = -Kx$.
