LINEAR FEEDBACK CONTROL - HOMEWORK 5

Assigned 2020.01.07. Submission deadline 2020.01.21 (for only those who want their homework to be marked).

Problems

1. Consider $\dot{x} = Ax$ and $y = Cx$, where

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}, \ C = \begin{bmatrix} 1 & 1 & 0 \end{bmatrix}$$

(1.1) Is (C, A) observable?

(1.2) Is (C, A) detectable?

2. Consider the cart-pendulum system in HOMEWORK 1 and the linearized system $\dot{x} = Ax + Bu$. For simplicity let $m = M = L = 1$. Suppose that we can observe only the *sum* of the position of the cart and the angle of the pendulum; namely the output y is

$$y = Cx = \begin{bmatrix} 1 & 0 & 1 & 0 \end{bmatrix} x$$

(2.1) Is (C, A) observable? Is (C, A) detectable?

(2.2) Compute a matrix L such that the eigenvalues of $A + LC$ are $-1, -2, -1 + j, -1 - j$.

(2.3) Find an estimator-based output-feedback controller (using L computed in (2.2) and F in HOMEWORK 4) to asymptotically stabilize the cart-pendulum system.

(2.4) What are the eigenvalues of the closed-loop system?

3. Consider again the linearized cart-pendulum system in HOMEWORK 1 and $y = Cx = [1\ 0\ 1\ 0]x$; for simplicity let $m = M = L = 1$. Design an output-feedback (error-feedback) controller such that the position of the cart asymptotically tracks a constant reference signal.

4. Consider a plant $\dot{x} = u$ ($x, u \in \mathbb{R}$), a ramp reference signal and a sinusoid disturbance signal with frequency 2rad/s. Design a state-feedback controller such that the state x tracks the ramp reference while rejects the sinusoid disturbance.

Bonus problem (Matlab)

In this homework, we introduce the following two Matlab functions related to observability.

(i) Let A, C be matrices of suitable sizes. obsv(A, C) computes the observability matrix of the pair (C, A).

(ii) Let A, B, C be matrices of suitable sizes. [$\tilde{A}$,$\tilde{B}$,$\tilde{C}$,V]=obsvf(A, B, C) decomposes the system given by (A, B, C) into observable and unobservable parts given by $(\tilde{A}, \tilde{B}, \tilde{C})$, and V is the invertible matrix for the corresponding similarity transformation.
